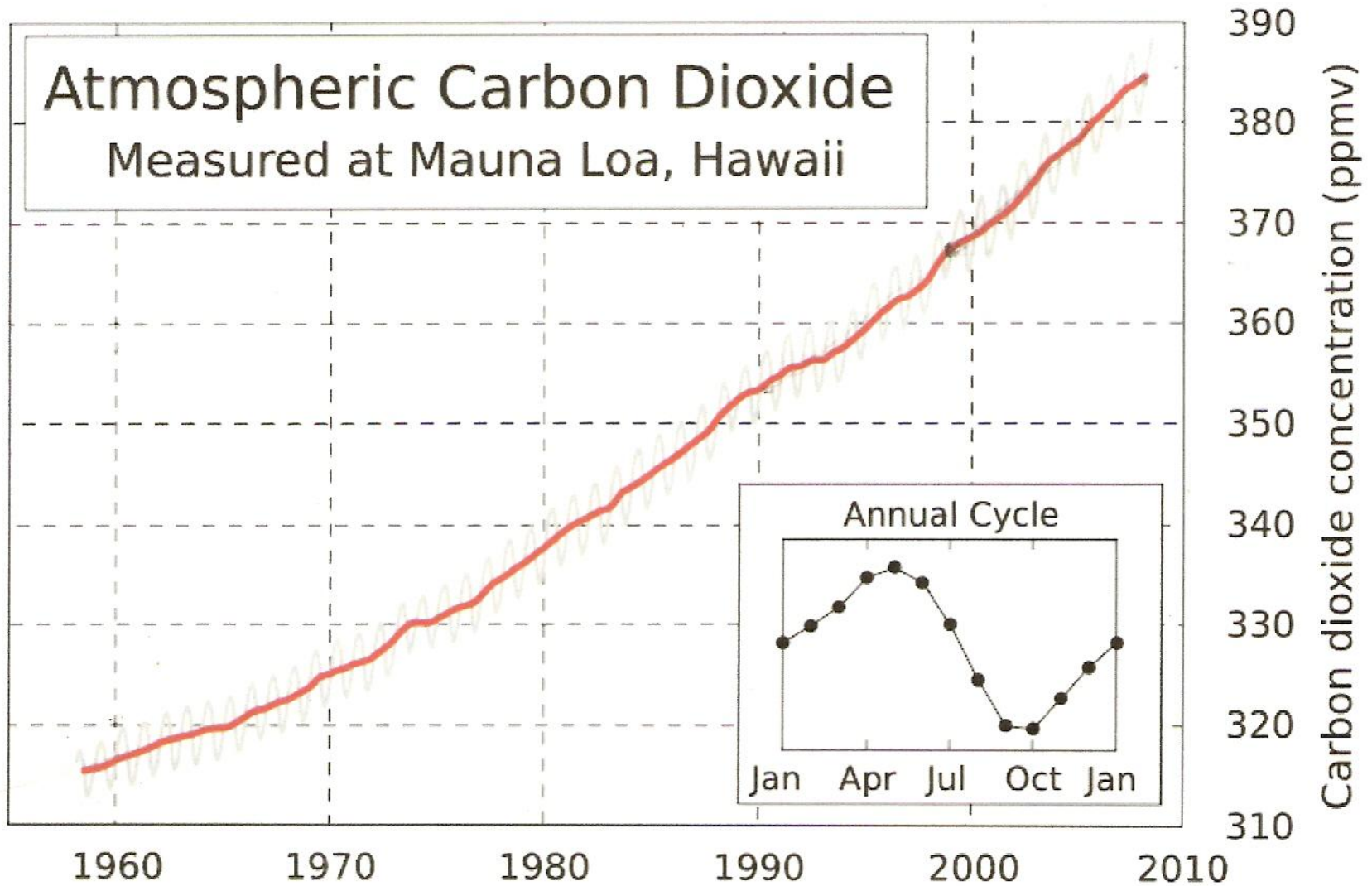


1. According to the U.S. DOE, each year humans are producing **28 billion metric tons** (1 metric ton = 2200 lbs) of CO₂ through the combustion of fossil fuels (oil, coal and natural gas).

2. Since the late 1950's spectroscopists at the Mauna Loa observatory have been collecting data on the concentration of CO₂ in the atmosphere. The plot of these data, as ppm vs. time, is known as the Keeling Curve.

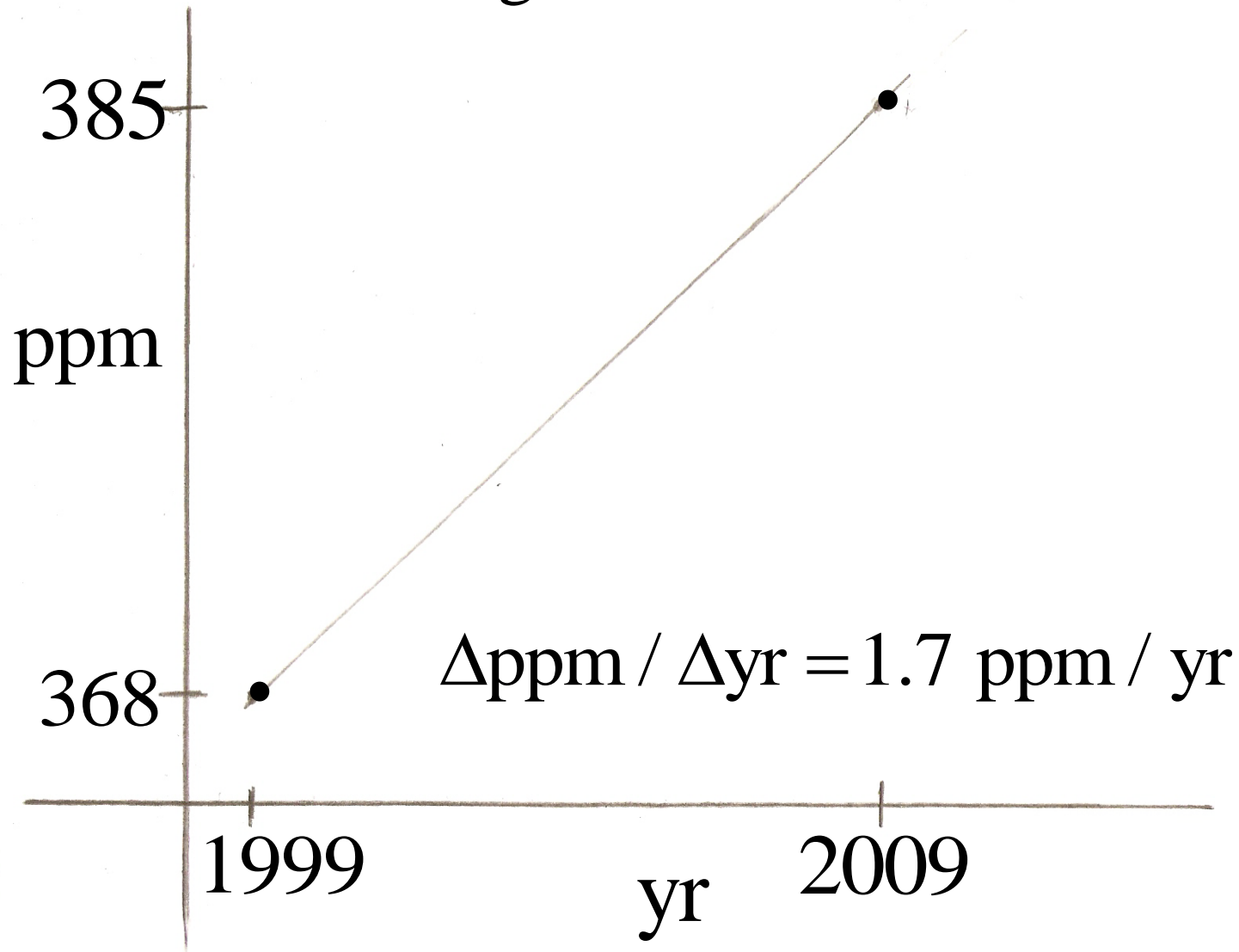
- File history
- File links

The Keeling Curve



Mauna_Loa_Carbon_Dioxide-en.svg (SVG file, nominally 850 × 547 pixels, file size: 16 KB)

Keeling Curve 1999 to Present



Thus CO₂ in the atmosphere is currently increasing by 1.7 ppm/yr. To compare this with the 28 billion metric tons per year produced by humans we need to determine how many moles this is and then multiply by the molecular weight of CO₂.

The atmospheric pressure on the earth is the force produced by the mass of air in the atmosphere divided by the surface area of the earth. Therefore to determine the mass of the atmosphere we need only multiply the surface area of the earth times atmospheric pressure:

$$\text{surface area} \times \text{atmospheric pressure} = m_{\text{air}}$$

$$5.11 \times 10^8 \text{ km}^2 \times 10^6 \text{ m}^2 / \text{km}^2 \times 1.01 \times 10^5 \text{ N/m}^2 / (9.8 \text{ N/kg}) = \\ 5.27 \times 10^{18} \text{ kg}$$

The mass of air divided by the molecular weight of air (.0288 kg/mole) equals the total number of moles of air:

$$\begin{aligned} m_{\text{air}} / M_{\text{air}} &= 5.27 \times 10^{18} \text{ kg} / 0.029 \text{ kg/mole} \\ &= 1.82 \times 10^{20} \text{ moles} \end{aligned}$$

Moles of air times the entering fraction of CO₂ equals moles of CO₂ entering from all sources:

$$n_{\text{air}} \times 1.7 \times 10^{-6} = 3.09 \times 10^{14} \text{ moles/yr}$$

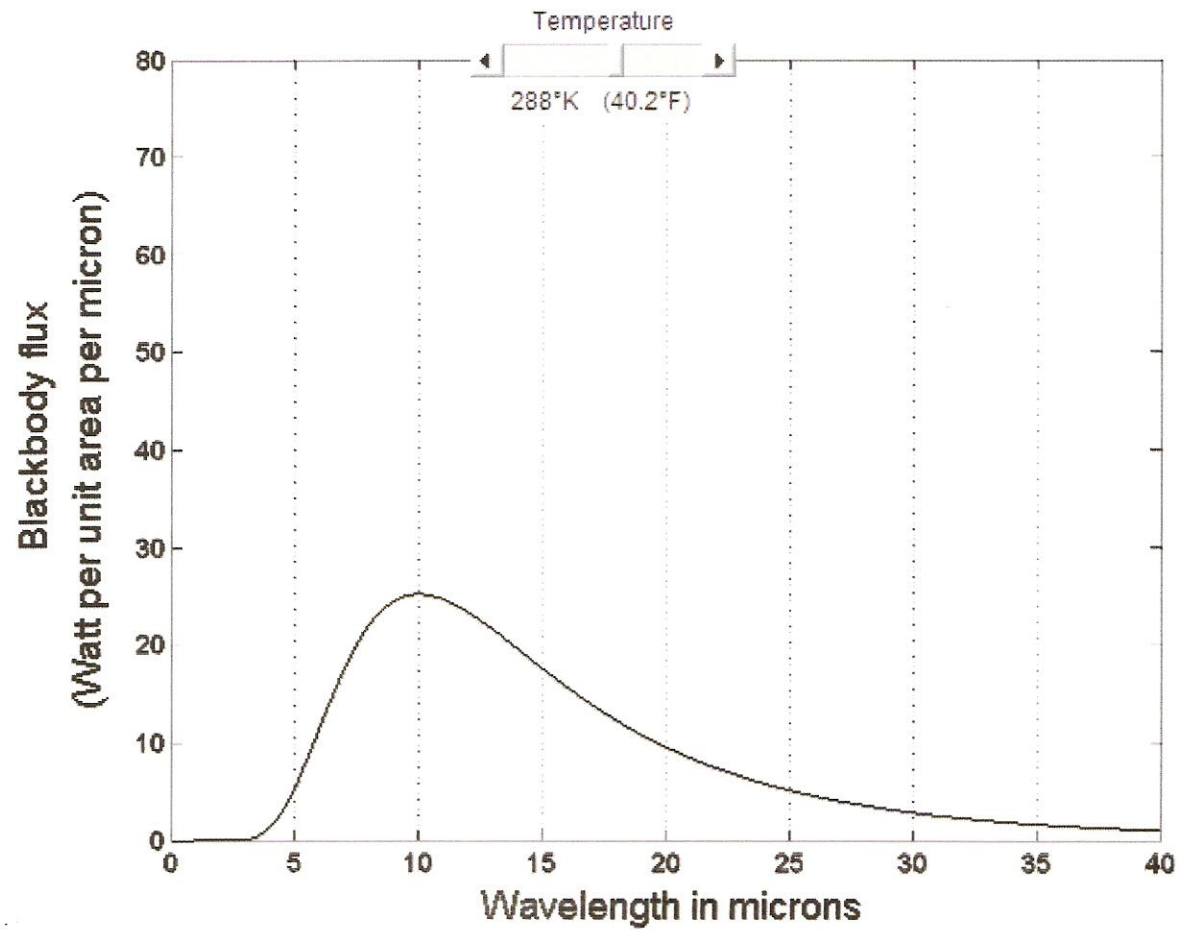
Finally, moles of CO₂ times the molecular weight of CO₂ equals the total (including all sources and sinks) mass of CO₂ entering the atmosphere per year:

$$\begin{aligned}\Delta n_{\text{CO}_2} / \Delta \text{yr} \times M_{\text{CO}_2} &= 3.09 \times 10^{14} \text{ moles/yr} \times 0.044 \text{ kg/mole} \\ &= 1.36 \times 10^{13} \text{ kg/yr} \\ &= 13.6 \text{ billion metric tons per year}\end{aligned}$$

Thus there is a net addition to the atmosphere of **13.6 billion metric tons** of CO₂ per year as a result of all sources and sinks - compare this with the **28 billion metric tons** produced by humans burning fossil fuels.

4. What does the CO₂ in the atmosphere do?

Among other affects it alters the earth's average temperature through interaction with the infrared radiation emitted by the earth. First consider the nature of this radiation. Equilibrium radiation is characterized by a Planck distribution and this distribution is generally assumed for the infrared emitted by the earth. The next slide shows a graph of a Planck distribution i.e the flux (energy per unit area per unit time) per unit wavelength versus wavelength for the current average temperature of the earth (288K).



To a good approximation the distribution of energy in the radiation from the sun also follows a Planck curve - one characterized by a temperature of about 5780 K at the surface of the sun. This radiation spreads out as the distance from the sun increases so the flux decreases. When this radiation strikes the earth on the average 0.3 (called the albedo) is reflected. The combined result is that the average net flux from the sun at the earth's surface is 240 watts m^{-2} .

5. What is the fate of the energy absorbed by the earth?

If the earth at some time and place is emitting more or less energy than it absorbs the earth's temperature changes until the radiative energy emitted to the cosmos equals the incoming energy from the sun. This is called a steady-state and the temperature is the steady-state emission temperature. These steady-state temperatures, when averaged over the whole earth and a year, would yield the earth's average steady-state emission temperature which is assumed (and the assumption is confirmed by measurement and averaging) to be the same as the steady-state temperature calculated for the known incoming flux of solar radiation.

Thus knowing the incoming flux we can calculate the earth's average steady-state temperature, barring some other source of energy at the earth. The Stefan Boltzmann Law relates the energy flux (F) of equilibrium radiation to T:

$$F = \sigma \times T^4$$

Using this law we can calculate the T at which the flux leaving the earth equals the nonreflected inward flux from the sun:

$$5.67 \times 10^{-8} \times T^4 = 240 \text{ watts m}^{-2}$$

yielding $T = 255 \text{ K}$ which is close to 0°F

Thus, were it not for some other effect (i.e. the greenhouse gas effect), when the earth reached a steady state with the radiation from the sun, i.e. when:

$$\text{energy in} = \text{energy out},$$

the average temperature of the earth's surface would be about 0° F. In reality the average temperature is 288 K (59° F). This is because greenhouse gases redirect some of the outgoing infrared radiation back to the earth's surface.

At the top of the atmosphere (to space):

$$\uparrow F^g(\text{TOA}) + F^{\text{atm}}(\text{TOA}) = 240 \text{ Watts/m}^2$$

At the earth's surface:

$$\downarrow F^{\text{sun}}(0) = 240 \text{ Watts/m}^2 \text{ from sun (after reflection)}$$

$$\downarrow F^{\text{atm}}(0) = 140 \text{ Watts/m}^2 \text{ from atmosphere}$$

$$\uparrow F^g(0) = 380 \text{ Watts/m}^2 \text{ from earth}$$

Remember the blue arrow

$$\text{Note that: } \downarrow = \uparrow$$
$$\uparrow - \downarrow = \uparrow$$

140 w m^{-2} of the outgoing infrared is redirected downward in the atmosphere by greenhouse gases (polyatomic gases with vibrational energies in the infrared region). H_2O is one such greenhouse gas. Another is CO_2 . CH_4 and O_3 are also. This redirected radiant energy is represented by the **blue arrow**.

6. What is the role of CO₂ in the redirection of the 140 w m⁻² back to the earth's surface?

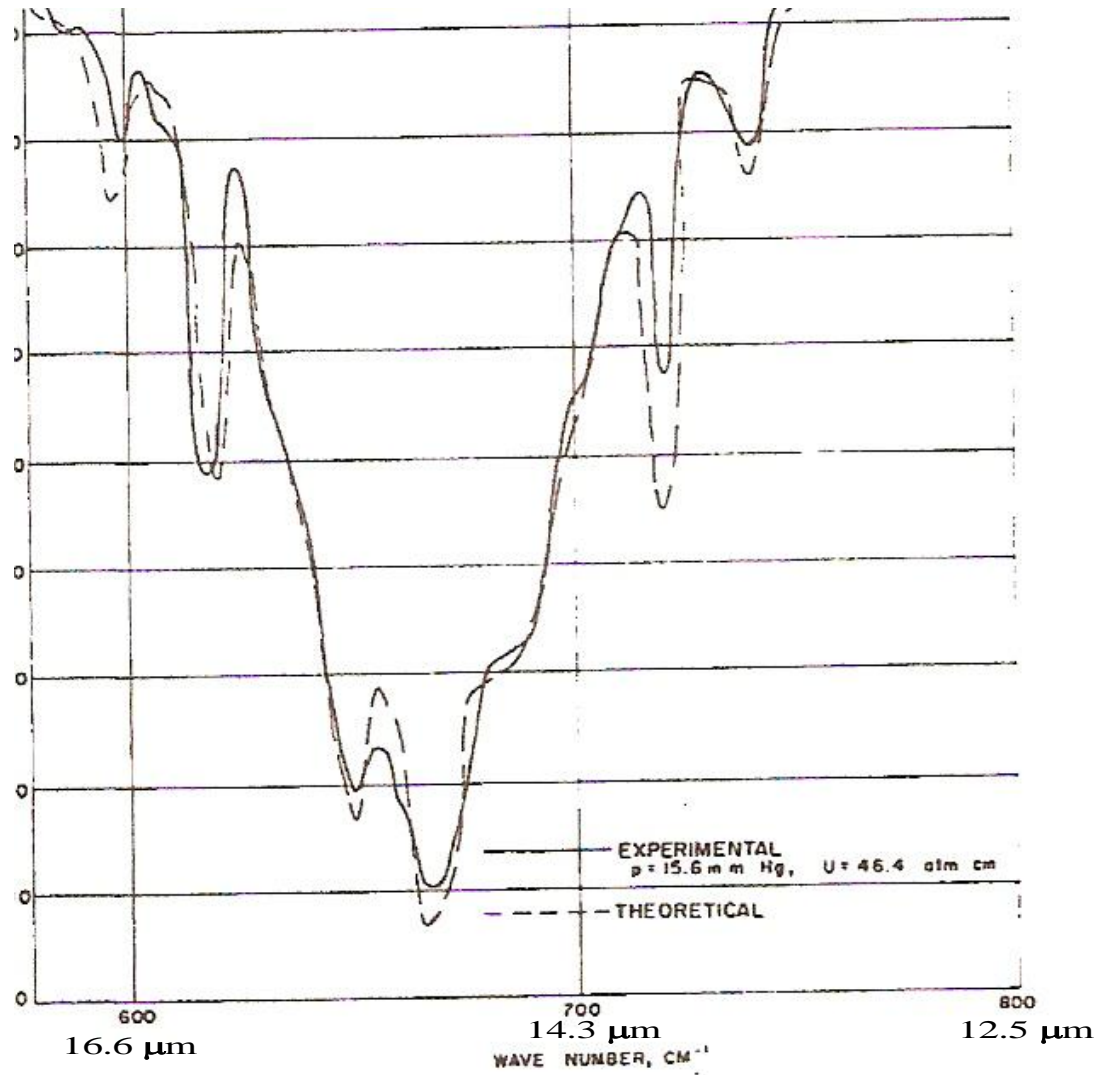
As everyone knows the climatologists have concluded that increases in the the **blue arrow** (greenhouse effect) effect are largely responsible for the currently observed warming of the earth, and that anthropogenic CO₂ makes a significant contribution to this effect. My goal here is to consider the basic physical chemistry underlying their calculations in order to determine whether or not basic chemcial physics leads to the same conclusion. I am certain that this has been done many times and that the results are well known to climatologists. I am also certain that these results are not widely known outside of this community.

The states of of CO₂ important in this discussion are the vibrational (separated by energies in the infrared region) and the rotational (separated by energies considerably lower than those separating the vibrational states) states of CO₂.

CO_2 is a linear molecule: $\text{O}=\text{C}=\text{O}$ and among its motions is the "asymmetric stretching motion" that absorbs infrared radiation with wavelengths in the vicinity of 15×10^{-6} meters (15 μm or 15 microns). In what follows the region from 9.3 μm to 19 μm will be denoted by $\Delta\lambda$. The reason it is necessary to designate a region is because each vibrational state (the transitions between adjacent vibrational states are the only ones important here) has associated with it rotational states such that transitions of interest are from one of the rotational states associated with the lowest vibrational state to one of the many that are associated with the adjacent vibrational state.

The following slide shows a calculated and observed absorption spectrum for CO₂ in the $\Delta\lambda$ interval from Stull, Wyatt, and Plaas, Applied Optics, Vol. 3 No. 2, p250 (1954), whose data will be used in a later calculation of the contribution of CO₂ to the greenhouse effect. Note the agreement between the observed and calculated spectra. Also note that there is more than one peak. Most of the peaks arise because of contributions from rotational states.

Theoretical and Calculated Absorbance of CO₂ in the Infrared



. 1. A comparison of the theoretical calculations of the transmittance with the experimental measurements of Burch *et al.*⁶ for a pressure of 15.6 mm Hg and 46.4 atm-cm of CO₂.

I will assume that the rotational-vibration transitions of CO₂ that are important in the atmosphere occur between 9.3 and 19 μm. This assumption is necessary because I am limited by the data available to me, but it is clear that the calculated effect would be increased, although not significantly, if the region were increased. The energy emitted in this region is found by taking the ratio of the total area under the Planck curve for 288 K to the area under the curve within the region. The result is that 53.1% of the radiation emitted by the earth is in the region of the chosen CO₂ rot-vib absorption lines.

The 53.1 % is obtained from:

$$\int_{2.89}^{5.92} u^3 / (e^u - 1) du / \int_0^{\infty} u^3 / (e^u - 1) du = 0.531$$

where $u = hc / \lambda kT$. This technique will be used again in a later slide.

A SIMPLE EXAMPLE OF A PLANCK DISTRIBUTION

The energy separating the ground and first asymmetric stretching states of CO_2 is 1.32×10^{-20} Joules. For these two states at equilibrium at a given temperature T :

$$[\text{CO}_2^*] / [\text{CO}_2] = e^{-960/T}.$$

If $[\text{CO}_2^*] / [\text{CO}_2] > e^{-960/T}$ then $\text{CO}_2^* \rightarrow \text{CO}_2 + \text{photon}$.

If $[\text{CO}_2^*] / [\text{CO}_2] < e^{-960/T}$ then $\text{CO}_2 + \text{photon} \rightarrow \text{CO}_2^*$

Thus the infrared radiation in the $\Delta\lambda$ interval emitted by the earth interacts with (is absorbed and reemitted by) CO_2 as it passes through the atmosphere.

The absorption of the infrared, because of the excitation of CO₂ from vib-rot states into vib-rot states, results in a decrease in the radiant flux that reaches the top of the atmosphere and originates from the ground. This decrease is, in the simplest case of a single linear path of a flux from a single source of nearly monochromatic radiation, given by:

$$dF = -kPFd\ell,$$

where dF is the change in flux, P is the pressure of the absorber, $d\ell$ is the path length interval and k is called the absorption coefficient. When integrated this gives Beer's Law:

$$\ln(F/F_0) = -kP\ell$$

↓ Beer's Law for linear absorption of radiation can be written:

$$\tau = e^{-u}$$

where τ is the linear transmittance and $u = kP\ell$ is the linear absorbance

However the case of radiation in the atmosphere has several complications:

1. the source is a band of energies and is in no sense monochromatic,
2. all directions (not just a single straight line) are of interest, and
3. the radiation that is reemitted is of interest.

Radiation emitted by the earth passes through the atmosphere interacting with the greenhouse gases up to the top of the atmosphere (TOA) where it passes into the cosmos. First consider the flux (radiant energy per unit area per unit time) corresponding to a single wavelength (λ) and proceeding along a line that makes the angle $\theta = \arccos \mu$ with the vertical. Later we will sum over the wavelengths in $\Delta\lambda$ and over the directions of an upward hemisphere. This flux is partially absorbed by the CO_2 (this absorption is characterized by an absorption coefficient, k_λ , by the amount of carbon dioxide in the path, P_{CO_2} , and by the length of the path, dz/μ (change in height divided by $\cos\theta$)). Furthermore additional radiation, B_λ , is emitted in all directions by CO_2 at z .

These thoughts lead to the basic equation relating the change in flux, $dF_{\lambda,\theta}$, the change in path length, dz/μ , the partial pressure of CO_2 , P_{CO_2} , and the absorption coefficient, k_λ , to the flux, $F_{\lambda,\theta}$, and the flux originating from the atmosphere, B_λ , all at height z as the radiation passes through the atmosphere.

$$dF_{\lambda,\theta} = -(F_{\lambda,\theta} - B_\lambda)(k_\lambda P_{\text{CO}_2})(dz / \mu)$$

The change in absorbance is:

$$du_{\lambda} = -k_{\lambda} P_{\text{CO}_2} dz$$

so the basic equation is,

$$\mu dF_{\lambda,\theta} / du_{\lambda} = F_{\lambda,\theta} - B_{\lambda}.$$

In what follows fluxes (W m^{-2}) at 0 (ground level) and at TOA (the top of the atmosphere) will be designated $F(0)$ and $F(\text{TOA})$, respectively. The origin of the flux will be designated by a superscript, e.g. $F^g(0)$ designates the flux at the ground level from the earth and $F^{\text{atm}}(\text{TOA})$ or $F^{\text{atm}}(0)$ are fluxes originating in the atmosphere.

A subscript, as in $F_{\lambda}^{\text{atm}}(0)$, will designate the flux at the given wavelength or, in the case of λ_i , a small interval of wavelengths centered on λ_i . A subscript, $\Delta\lambda$, as in $F_{\Delta\lambda}^g(0)$, will indicate a flux that has been integrated or summed over the whole $\Delta\lambda$ interval.

Multiplying through the basic equation by $e^{-\mu^{-1}u_\lambda}$, where u_λ is the linear absorbance :

$$\mu \times e^{-\mu^{-1}u_\lambda} dF_{\lambda,\theta} / du_\lambda - e^{-\mu^{-1}u_\lambda} F_{\lambda,\theta} = -e^{-\mu^{-1}u_\lambda} B_\lambda.$$

Note that the left hand side is $\mu \times d[e^{-\mu^{-1}u_\lambda} F_{\lambda,\theta}] / du_\lambda$. Therefore, integrating from the ground level at which:

$$z=0, u_\lambda = u_\lambda(0), F_{\lambda,\theta} = F_{\lambda,\theta}^g(0) = B_\lambda(0)$$

to the top of the atmosphere, TOA, where:

$$z = \infty, F_{\lambda,\theta} = F_{\lambda,\theta}^g(\text{TOA}), u_\lambda = 0,$$

$$F_{\lambda,\theta}^g(\text{TOA}) - e^{-\mu^{-1}u_\lambda(0)} F_{\lambda,\theta}^g(0) = - \int_{u_\lambda(0)}^0 B_\lambda e^{-\mu^{-1}u_\lambda} \mu^{-1} du_{\lambda,\ell}.$$

Now, to get the total upward λ flux at TOA, integrate over the upward hemisphere;

$$F_{\lambda}(\text{TOA}) = F_{\lambda}^{\text{g}}(0) \times 2 \times \int_0^1 e^{-\mu^{-1}u_{\lambda}(0)} \mu d\mu + 2 \times \int_0^1 \int_0^{u_{\lambda}(0)} B_{\lambda,z} e^{-\mu^{-1}u_{\lambda}} d\mu du_{\lambda}.$$

$$= F_{\lambda}^{\text{g}}(\text{TOA}) + F_{\lambda}^{\text{atm}}(\text{TOA})$$

furthermore $F_{\lambda}^{\text{g}}(0) - F_{\lambda}^{\text{g}}(\text{TOA}) = F_{\lambda}^{\text{atm}}(0) + F_{\lambda}^{\text{atm}}(\text{TOA}) = 2F_{\lambda}^{\text{atm}}(0)$

thus $F_{\lambda}^{\text{g}}(0) - F_{\lambda}^{\text{g}}(0) \times 2 \times \int_0^1 e^{-\mu^{-1}u_{\lambda}(0)} \mu d\mu = 2F_{\lambda}^{\text{atm}}(0)$

i.e. the flux that originates from the ground but is scattered is twice the flux that returns to the earth because half the scattered flux returns to the earth and half leaves through the top of the atmosphere

Introducing the notation:

$$\langle \tau_{\lambda}^g(0) \rangle = 2 \int_0^1 e^{-\mu^{-1}u_{\lambda}(0)} \mu d\mu$$

for the diffuse transmissivity and integrating the quantity

$$F_{\lambda}^g(\text{TOA}) = F_{\lambda}^g(0) \times 2 \times \int_0^1 e^{-\mu^{-1}u_{\lambda}(0)} \mu d\mu = F_{\lambda}^g(0) \langle \tau_{\lambda}^g(0) \rangle$$

over $\Delta\lambda$ to get the flux at TOA originating from the ground over the $\Delta\lambda$ interval (9.3 to 19 μm) yields:

$$F_{\Delta\lambda}^g(\text{TOA}) = \int_{9.3\mu\text{m}}^{19\mu\text{m}} F_{\lambda}^g(0) \langle \tau_{\lambda}^g(0) \rangle d\lambda$$

where, as described previously, $F_{\lambda}^g(0)$ is the Planck flux with wavelength λ at the ground and originating from the earth.

Recall Planck's law: $F_{\Delta\lambda}^g(0) = \alpha \int_{u(9.3\mu\text{m})}^{u(19\mu\text{m})} u^3 / (e^u + 1) du$ with u

evaluated at the earth's average temperature. Divide the interval into 11 smaller intervals labeled by i (runs from 1 to 11) where:

$\lambda_1^{-1} = 55000 \text{ m}^{-1}$, $\lambda_2^{-1} = 60000 \text{ m}^{-1}$, $\lambda_3^{-1} = 65000 \text{ m}^{-1}$, etc

until $\lambda_{11}^{-1} = 1025000 \text{ m}^{-1}$. (the intervals are expressed in

as in terms of wave numbers (λ^{-1}) because the

absorption data are published vs. λ^{-1}). Define \mathfrak{G}_i to be that

fraction of the flux in the interval between 5.25 and $10.75 \times 10^4 \text{ m}^{-1}$

(9.3 and $19 \mu\text{m}$) that falls between λ_{i-1}^{-1} and λ_i^{-1}

$$\mathfrak{G}_i = \alpha \left[\int_{u(\lambda_{i-1}^{-1})}^{u(\lambda_i^{-1})} u^3 / (e^u - 1) du \right] / F_{\Delta\lambda}^g(0) = F_{\lambda_i}^g(0) / F_{\Delta\lambda}^g(0)$$

The slide before the last ended with:

$$F_{\Delta\lambda}^g(\text{TOA}) = \int_{12\mu\text{m}}^{18\mu\text{m}} F_{\lambda}^g(0) \langle \tau_{\lambda}(0) \rangle d\lambda$$

Replacing the integral over λ by a sum:

$$F_{\Delta\lambda}^g(\text{TOA}) = \sum_{i=1}^{11} F_{\lambda_i}^g(0) \langle \tau_{\lambda_i}(0) \rangle$$

and, introducing $\mathfrak{G}_i = F_{\lambda_i}^g(0) / F_{\Delta\lambda}^g(0)$,

$$F_{\Delta\lambda}^g(\text{TOA}) = F_{\Delta\lambda}^g(0) \sum_{i=1}^{11} \mathfrak{G}_i \langle \tau_{\lambda_i}(0) \rangle$$

$\sum_{i=1}^{11} \vartheta_i \langle \tau_{\lambda_i}(0) \rangle$ is the Planck weighted diffuse transmissivity

for the $\Delta\lambda$ interval from the ground to TOA. The value for

$F_{\Delta\lambda}^g(\text{TOA})$ is the product of this quantity with $F_{\Delta\lambda}^g(0)$. Thus the problem of relating the flux leaving the earth to that originating

from the earth is solved when $\sum_{i=1}^{11} \vartheta_i \langle \tau_{\lambda_i}(0) \rangle$ is known.

Values for the Planck weighted transmissivities are obtained below using the in-line transmissivity values reported by Stull, Wyatt, and Plass, Applied Optics, Vol.3, No.2, p243 (1964) and given for four λ_i^{-1} values in the $\Delta\lambda$ interval in the table on the next slide. Values for the integral over a hemisphere

$$\langle \tau_{\lambda_i}(0) \rangle = \int_0^1 e^{-\mu^{-1} u_{\lambda_i}(0)} \mu d\mu$$

and for \mathfrak{G}_i were calculated by integration using the given u_{λ_i} values

Transmissivities

| cm^{-1} | 1000 atm-cm | 500 atm-cm | 200 atm-cm | 100 atm-cm | \mathfrak{g}_i |
|------------------|-------------|------------|------------|------------|------------------|
| 550 | 0.685 | 0.809 | 0.914 | 0.956 | 0.1104 |
| 600 | 0.023 | 0.063 | 0.165 | 0.282 | 0.1169 |
| 650 | 0 | 0 | 0 | 0.002 | 0.1138 |
| 700 | 0 | 0.003 | 0.013 | 0,038 | 0.1085 |
| 750 | 0.115 | 0.199 | 0.341 | 0.466 | 0.1017 |
| 800 | 0.705 | 0.824 | 0.929 | 0.959 | 0.09903 |
| 850 | 0.672 | 0.7775 | 0.875 | 0.927 | 0.08581 |
| 900 | 0.823 | 0.885 | 0.929 | 0.989 | 0.06921 |
| 950 | 0.909 | 0.949 | 0.978 | 0.994 | 0.06132 |
| 1000 | 0.948 | 0.972 | 0.988 | 0.994 | 0.06132 |
| 1050 | 0.775 | 0.856 | 0.927 | 0.960 | 0.05390 |

The products $\mathfrak{D}_i \langle \tau_{\lambda_i}(0) \rangle$ and the sums of these over the $\Delta\lambda$ interval are then given for the four λ_i^{-1} on the following slide.

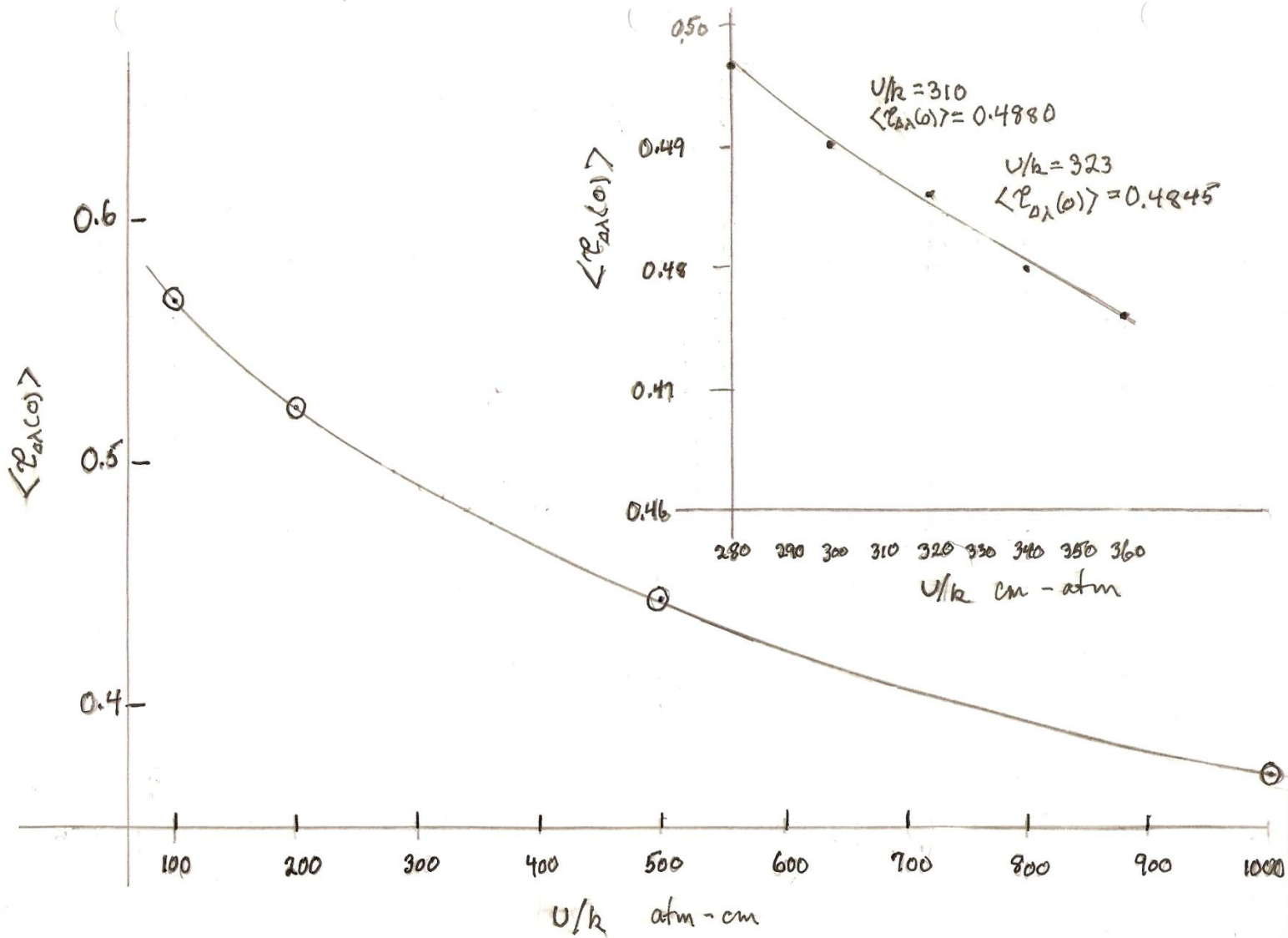
$$\vartheta_i \langle \tau_{\lambda_i}^g(0) \rangle$$

| cm ⁻¹ | 1000 atm-cm | 500 atm-cm | 200 atm-cm | 100 atm-cm |
|------------------|---------------|---------------|---------------|---------------|
| 550 | 0.0624 | 0.0810 | 0.0994 | 0.1014 |
| 600 | 0.0008 | 0.0028 | 0.0090 | 0.0180 |
| 650 | 0 | 0 | 0 | 0 |
| 700 | 0 | 0 | 0.0004 | 0.0012 |
| 750 | 0.0050 | 0.0100 | 0.0201 | 0.0309 |
| 800 | 0.0550 | 0.0704 | 0.0850 | 0.0915 |
| 850 | 0.0443 | 0.0553 | 0.0674 | 0.0745 |
| 900 | 0.0550 | 0.0621 | 0.0675 | 0.0725 |
| 950 | 0.0581 | 0.0627 | 0.0663 | 0.0677 |
| 1000 | 0.0555 | 0.0581 | 0.0599 | 0.0606 |
| 1050 | 0.0347 | 0.0408 | 0.0468 | 0.0499 |
| Σ | 0.3709 | 0.4433 | 0.5221 | 0.5682 |

Our interest here is to find the broadband diffuse transmissivity of CO₂ :

$$\sum_{i=1}^{11} g_i \langle \tau_{\lambda_i}^g(0) \rangle = \langle \tau_{\Delta\lambda}^g(0) \rangle$$

appropriate to the intermediate values characteristic of the current (385 ppm) and 100 years hence (555 ppm) CO₂ concentrations in the total atmosphere. To find these we interpolate a plot of the four values obtained for the broadband diffuse transmissivity (shown on the next slide) to obtain values of u/k appropriate to the CO₂ in the atmosphere as found on the slide that follows the next one.



I obtained the optical depths for the atmosphere as follows.

By definition:

$$u(z) = \int_z^{\infty} -kPdz.$$

For CO₂ in the current atmosphere $P = P_{\text{air}} \times 3.85 \times 10^{-4}$,
and $P_{\text{air}} = P(0) \times e^{-Mgz/(8.314T)} = e^{-0,03395z/T}$, according to the
barometric formula. Thus, taking $k(T,P)$ constant:

$$u(0) = -k \times P_{\text{air}}(0) \times 3.85 \times 10^{-4} \times \int_0^{\infty} e^{-0.03395z/T} dz$$

Taking $P_{\text{air}}(0) = 1$ atm and $T = 288$ K at $z=0$ and decreasing by
6.5 K km⁻¹ from 0 to 10 km, constant from 10 to 15 km and
increasing by 1.1 K km⁻¹ from 15 to 50 km integration yields:

$$u(0)/k = 3.25 \text{ atm-m (or 325 atm-cm),}$$

For 555 ppm $u(0)/k = 469$ atm-cm.

The interpolated values found for:

$$\sum_{i=1}^{11} \vartheta_i \langle \tau_{\lambda_i}^g(0) \rangle = \langle \tau_{\Delta\lambda}^g \rangle$$

are:

$$\text{at 385 ppm } \langle \tau_{\Delta\lambda}^g(0) \rangle = 0.486$$

$$\text{at 555 ppm } \langle \tau_{\Delta\lambda}^g(0) \rangle = 0.454$$

According to what was found above the average contribution of CO₂ to the **blue arrow** flux is:

$$F_{\Delta\lambda}^{\text{atm}}(0) = 1/2 \times 0.531 \times F_{\text{tot}}^{\text{g}}(0) \times (1 - \langle \tau_{\Delta\lambda}^{\text{g}}(0) \rangle)$$

This equation says that one half the radiation scattered by CO₂ returns to the earth. Accordingly, using the Planck weighted transmissivity values, the **blue arrow** effects of CO₂ at 385 and 555 ppm are:

$$F_{\text{tot}}^{\text{atm},385}(0) = 0.1365 \times F_{\text{tot}}^{\text{g},385}(0),$$

and

$$F_{\text{tot}}^{\text{atm},555}(0) = 0.1450 \times F_{\text{tot}}^{\text{g},555}(0).$$

The increase in the steady state flux to the ground level from the atmosphere when the CO₂ increases from 385 to 555 ppm is matched by an increased Planck flux from the ground when a steady state is reached at each partial pressure:

$$\Delta F_{\Delta\lambda}^{\text{atm}}(0) = \Delta F_{\text{tot}}^{\text{g}}(0).$$

Therefore

$$F_{\text{tot}}^{\text{g},555}(0) - F_{\text{tot}}^{\text{g},385}(0) = 0.1450 \times F_{\text{tot}}^{\text{g},555} - 0.1365 \times F_{\text{tot}}^{\text{g},385}$$

Dividing by $F_{\text{tot}}^{\text{g},385}(0)$ and collecting terms:

$$F_{\text{tot}}^{\text{g},555} / F_{\text{tot}}^{\text{g},385} (1 - 0.1450) = (1 - 0.1365)$$

or

$$F_{\text{tot}}^{\text{g},555} / F_{\text{tot}}^{\text{g},385} = 1.0099$$

Using the Stefan-Boltzmann law

$$\begin{aligned} F_{\text{tot}}^{\text{g},555}(0) / F_{\text{tot}}^{\text{g},385}(0) &= (T + \delta T)^4 / T^4 \\ &= (T^4 + 4T^3\delta T) / T^4, \end{aligned}$$

$$F_{\text{tot}}^{\text{g},555}(0) / F_{\text{tot}}^{\text{g},385}(0) = 1 + 4\delta T/T,$$

$$4\delta T/T = 0.0099,$$

and,

$$\delta T = 0.7 \text{ K}.$$

7. Comments on the result:

A. According to the IPCC the average surface temperature of the earth increased by $0.74 \pm .18\text{C}^{\circ}$ between 1905 and 2005. The 100 year increase found here (0.7 K) shows that there is solid science (of which the climatologists are thoroughly aware) supporting their conclusion that human production of CO_2 plays an important role in global warming.

B. The increase in temperature found here is a lower bound because: 1. it is based upon the assumption that the rate of production of CO_2 will not increase. As has been widely discussed substantial increases are expected in China, India, and in the developing world. 2. feedbacks have been ignored e.g. i. melting of arctic ice, ii. release of CO_2 from the earth and ocean due to increased temperature, iii. release of CH_4 from permafrost and ocean floor, iii. changes in the concentration of water in the atmosphere

